

# Taxonomy of Theoretical Temporal-Graph Problem Families

Manuel Sorge · Codex

## Executive Summary

This report maps the main problem families studied in theoretical temporal graphs, with an emphasis on algorithmic complexity, approximation, parameterized complexity, online analysis, and distributed protocols. The scope is time-evolving graphs represented as time-labeled edges (contact sequences), snapshot sequences, interval/link-stream models, and periodic variants. The central finding is that most of the mature theory is organized around **temporal path semantics** and **connectivity over time**, while several fast-growing subareas (realization, interdiction, counting/enumeration) are still structurally fragmented.

## Core Taxonomy Axes

The literature is most coherently classified along five axes:

1. **Input-time model**: edge-time tuples, snapshots, intervals/link streams, periodic schedules [55, 18].
2. **Journey semantics**: strict vs non-strict temporal paths, waiting policy, directedness, and additional constraints such as simple/proper/happy paths [15, 26].
3. **Objective type**: feasibility/decision, optimization, counting, enumeration, and realization/inference [42, 27].
4. **Complexity regime**: polynomial-time solvable fragments, NP-hardness/APX-hardness, FPT/W-hardness, kernelization, and approximation tradeoffs [4, 3, 51].
5. **Temporal uncertainty regime**: offline (full timeline known), online (future unknown), adversarial/distributed models [31, 45].

## Core vs Niche (evidence-based)

### Core (densest and most mature):

- Temporal reachability and path semantics [42, 15].
- Temporal shortest-path objective variants (foremost/fastest/shortest/latest-departure) [12, 59].
- Temporal connectivity/substructures: temporally connected spanning subgraphs, separators/cuts, and spanners [4, 60, 23].
- Temporal exploration/traversal [29, 34].
- Sliding-window/temporal analogues of NP-hard problems, especially temporal vertex cover and temporal matching [3, 51].

### Niche but accelerating:

- Reachability-graph recognition/realization and related inference/reconstruction [30, 16, 48].
- Interdiction objectives for temporal routing [10].
- Counting/enumeration (e.g., counting temporal paths, enumerating temporal spanners) [27, 47].
- Temporal connectivity augmentation and labeling design [9, 43].

## Main Structural Message

Temporal-graph theory is not just “static graph problems + time.” The key break is that different temporal path semantics are genuinely non-equivalent and can invert complexity status, approximation behavior, and

parameterized tractability [12, 20, 15]. This is why shortest-path problems split into multiple objectives and why connectivity has many incompatible formulations. A useful unified view is:

- **Path semantics layer** determines what constitutes causality and feasibility.
- **Optimization layer** selects one of several incomparable objectives.
- **Global structure layer** asks for sparse certificates, cuts/separators, or augmentations preserving temporal connectivity.
- **Algorithmic regime layer** (offline/online/distributed/parameterized) determines what guarantees are realistic.

An alternative and often cleaner organization is to classify by **problem primitive first** (feasibility, optimization, design, traversal, synthesis, counting), then attach temporal constraints (strictness, waiting, uncertainty, structure) as overlays.

### Maturity Snapshot by Family

- **Reachability/closure**: foundational and currently expanding through recognition/realization and definitional refinement [42, 30].
- **Shortest temporal routing**: mature on objective taxonomy; active on constrained waiting and interdiction [12, 10].
- **Connectivity/substructures**: rich hardness + approximation + sparsification literature; separators and augmentation now tie graph classes to algorithms [4, 60, 9].
- **Exploration**: strong offline hardness/approximation base with growing parameterized and online variants [29, 34].
- **Multistage/timeline optimization**: broadening beyond vertex cover to matching/path/covering formulations; definitions still fragmented [3, 38, 25].
- **Counting/enumeration**: comparatively young; complexity-theoretic foundations are appearing recently [27, 47].
- **Random/probabilistic temporal graphs**: emerging theory line with sharp threshold results and strong typical-case sparsification behavior [24, 7].

### Method and Inclusion Protocol

This report follows citation snowballing from seed theory surveys/foundations, then forward/backward expansion by problem family:

1. Seed baseline: [55, 42, 18].
2. Family anchoring seeds: reachability semantics [15], exploration [29], temporal vertex cover [3], spanners/augmentation [23, 9], shortest-path variants [12].
3. Expansion rule: include primary theory papers (conference/journal) that establish complexity, approximation, parameterized, online, or distributed results; de-emphasize purely empirical measures unless tied to formal temporal-graph statements.

### Coverage Map and Boundaries

This taxonomy is intended to be comprehensive for the main algorithmic-theory lines, but not for all temporal-network research.

- **Included as core**: reachability/journeys; temporal routing metrics; connectivity design/cuts/spanners; exploration; multistage timeline optimization; counting/enumeration; online/distributed models.
- **Included as frontier**: realization/inference from path constraints; temporal interdiction; random temporal-graph thresholds.

- **Intentionally de-emphasized:** empirical centrality/motif studies and application-driven forecasting unless they yield formal problem definitions and complexity/algorithmic results.

### **Taxonomy Table (Main Artifact)**

The overview below is placed in landscape because the seven-axis classification is not legible in portrait. The detailed family sections that follow keep the full prose discussion.

Problem family	Canonical names / synonyms	Input model(s)	Path semantics / assumptions	Objective type	Representative results (complexity/approx/FPT)	Key citations
<b>A. Paths, reachability, closure, realization</b>	Temporal reachability; temporal transitive closure; temporal components; reachability-graph recognition/realization	Time-labeled edges; snapshots; interval models; directed or undirected	Strict vs non-strict; waiting allowed/forbidden; directedness; simple/proper/happy variants	Decision; max reachability; realization/inference	Foundational causality/connectivity model. Strict separations between path-regime definitions. Realization shows mixed tractability and new exact/hardness boundaries.	[42, 15, 26, 30, 16, 44, 48]
<b>B. Shortest-path variants and routing objectives</b>	Foremost; fastest; shortest; latest-departure; temporal routing; temporal path interdiction	Contact sequences; snapshots; intervals; weighted temporal edges	Strict/non-strict; waiting constraints (including restless); directed or undirected	Optimization; decision; interdiction; robustness	Foremost/fastest/shortest are inequivalent and need different algorithms. Waiting restrictions create sharp complexity changes. Interdiction is polytime for some metrics and hard for others.	[12, 58, 59, 19, 20, 10, 40]
<b>C. Connectivity and substructures</b>	Minimum temporally connected subgraph; temporal spanning subgraph; tree; arborescence; temporal cuts/separators; temporal spanners; connectivity augmentation	Time-labeled edges; snapshots; interval temporal graphs	Usually strict reachability with waiting; directed or undirected; often all-pairs temporal connectivity	Optimization; approximation; parameterized decision; sparsification	MTC has strong hardness/approximability barriers and can require dense certificates. Temporal cliques admit significantly sparse spanners. Separator/cut problems have detailed complexity maps and FPT islands.	[4, 1, 23, 8, 60, 35, 57, 9, 11]
<b>D. Exploration / traversal / search</b>	Temporal graph exploration; temporal walk exploration; temporal edge exploration; temporal TSP variants	Primarily snapshots or edge-time labels; sometimes periodic models	Walk semantics (vertex revisit allowed); strict/non-strict; offline vs online	Optimization (arrival or makespan); approximation; online competitiveness; parameterized complexity	TGE is hard even on restricted classes. Non-strict and edge-deficient variants refine tractability boundaries. Parameterized exploration and edge exploration add FPT-style progress.	[28, 29, 31, 32, 33, 34, 13, 6]

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Problem family	Canonical names / synonyms	Input model(s)	Path semantics / assumptions	Objective type	Representative results (complexity/approx/FPT)	Key citations
<b>E. Multistage / timeline analogues of classic NP-hard problems</b>	Temporal Vertex Cover / Sliding-Window TVC / Timeline Cover; Multistage Vertex Cover; Multistage s-t Path; temporal matching; temporal coloring	Snapshot and timeline models with per-time-layer constraints; interval activity in some variants	Usually per-layer feasibility plus temporal consistency or smoothness constraints	Decision; optimization; approximation; FPT/XP	Sliding-window TVC has approximation and FPT results but remains hard in general. Temporal matching and temporal coloring add their own hardness plus algorithmic islands. Multistage path/cover models make quality-vs-stability tradeoffs explicit.	[2, 3, 39, 50, 51, 52, 53, 38, 36, 25]
<b>F. Counting and enumeration</b>	Counting temporal paths/walks; counting temporal centrality witnesses; enumeration of temporal spanners/substructures	Time-labeled edges; snapshots; interval models	Same journey semantics as the decision variants; strict/non-strict distinction often crucial	Counting (#P); parameterized counting; output-sensitive enumeration	Counting temporal paths now has explicit complexity-theoretic treatment. Enumeration of temporal spanners has recently received dedicated algorithms. Counting/enumeration remains much less developed than decision/optimization families.	[27, 47, 23, 8, 48]
<b>G. On-line/distributed temporal communication (cross-cutting)</b>	Temporal broadcast; convergecast; distributed computation in dynamic networks; online temporal routing/exploration	Dynamic network models; synchronous or asynchronous rounds; snapshot-like evolving graphs	Causal journeys; adversarial edge dynamics; unknown future in online variants	Protocol design; feasibility/lower bounds; competitive analysis	Dynamic-network distributed computation has strong lower-bound results. Broadcast objectives split by shortest/fastest/foremost criteria. Online exploration variants clarify the cost of uncertainty about future edge availability.	[45, 18, 17, 54, 31, 34]
<b>H. Random/probabilistic temporal graph theory</b>	Random simple temporal graphs; temporal Erdős-Rényi thresholds; probabilistic temporal connectivity	Random edge-order or permutation models over random or static supports	Usually strict temporal paths with random timestamps; a.a.s. guarantees	Threshold phenomena; extremal typical-case structure; probabilistic existence	Sharp thresholds separate pairwise, single-source, and all-pairs temporal reachability. Giant temporal components emerge at a sharp threshold distinct from static intuition. Typical-case temporal spanners can be near-optimal despite worst-case lower bounds.	[24, 7, 23, 4]

## Alternative Organization: Primitive-First Taxonomy

The family table above is objective-centric. A complementary organization that better isolates definitional effects is to map papers by **primitive** first, then apply temporal constraints as overlays.

Primitive (primary)	Static analogue	Temporalized question	Typical outputs	Representative families
Causal feasibility	Reachability / transitive closure	Is $v$ temporally reachable from $u$ under semantics $S$ ?	Decision, closure graph, components	A
Route quality	Shortest path	Optimize arrival/duration/hops/departure under time-respecting constraints	Optimal path/walk, bicriteria frontiers	B
Global connectivity design	Spanners / Steiner / cuts	Keep all-pairs temporal connectivity with minimal labels/edges or destroy it via separators	Sparse certificates, separators, augmentations	C
Traversal completion	Graph exploration / TSP / Euler tour	Visit all vertices/edges in minimum temporal makespan	Walk schedule, approximation bounds	D
Repeated-feasibility optimization	Covering and packing over time	Choose per-time solutions with window constraints and change penalties	Timeline solution sequence	E
Counting and enumeration	#P counting / output-sensitive listing	Count or enumerate temporal objects consistent with semantics $S$	# counts, enumerated families	F
Protocol under uncertainty	Distributed communication	Compute/broadcast in dynamic links with incomplete future information	Feasibility, lower bounds, competitiveness	G
Typical-case thresholds	Random graph phase transitions	When do temporal connectivity/substructure properties appear a.a.s.?	Sharp thresholds, probabilistic structural laws	H

### Constraint overlays (apply to all primitives)

- **Journey semantics:** strict/non-strict, waiting policy (including restless).
- **Input model:** contact sequence, snapshots, interval/link-stream, periodic.
- **Directionality:** directed vs undirected temporal edges/arcs.
- **Knowledge regime:** offline timeline, online revelation, adversarial dynamics.
- **Complexity regime:** classical complexity, approximation, parameterized, distributed lower bounds.

### Family A: Temporal Paths, Reachability, and Closure

**Why central.** Reachability is the base predicate from which temporal connectivity, components, path optimization, and distributed causal communication are derived [42, 55].

**First formalization and refinements.** Canonical formalization in this line starts with temporal connectivity/reachability in evolving networks [42], then branches into explicit path-regime taxonomies [15, 26] and realization/inference formulations [30, 16, 48].

#### Definition forks that change results.

- **strict vs non-strict** time monotonicity.
- **Waiting allowed vs forbidden/bounded** (restless).

- Directed vs undirected edge-time labels.
- Additional constraints on repeated vertices/edges captured by simple/proper/happy variants [15, 26].

These are not cosmetic forks: recent work proves strict separations between these path regimes [15].

#### **Canonical techniques.**

- Time expansion / layered DAG constructions for reachability and reductions.
- Closure graph constructions (reachability graphs) and realization by temporal labels.
- Structural parameters related to transitivity distance and graph classes [21].

#### **What is known.**

- Foundational models of evolving-graph reachability and connectivity are classical by now [42].
- Reachability-graph recognition/realization has become a dedicated line, with specialized algorithms and hardness boundaries [30, 16, 48].
- Temporal components are sensitive to the underlying reachability semantics [5, 15].

#### **Open gaps with evidence.**

- A unified complexity classification over all major path semantics (strict/non-strict + waiting + directedness + simplicity constraints) is still missing; current results are fragmented by model assumptions [15, 30].
- Realization/inference is expanding quickly, but many natural constraints (bounded label multiplicity, robustness/noise) are only partially mapped [48, 49].

## **Family B: Temporal Shortest-Path Variants and Routing**

**Why central.** In static graphs, shortest path is largely canonical. In temporal graphs, at least four inequivalent notions are standard: earliest-arrival (foremost), minimum-duration (fastest), minimum-hop/cost (shortest), and latest-departure [12, 59].

**First formalization and refinements.** The formal split into foremost/fastest/shortest is classical [12], with modern refinements for constrained waiting [20] and interdiction-oriented objectives [10].

#### **Definition forks that change results.**

- Objective selection (foremost vs fastest vs shortest) itself changes optimal solutions.
- Waiting policy (unbounded, bounded, forbidden) modifies feasibility and complexity [20].
- Edge traversal times / interval availability alter algorithm design [58, 40].

#### **Canonical techniques.**

- Earliest-arrival scans over time-sorted edge streams.
- Time-expanded graphs plus label-setting dynamic programming.
- Reduction to path problems with additional temporal-state dimensions (for waiting/restless variants).

#### **What is known.**

- Objective taxonomy and baseline algorithms are well established [12, 59].

- Constrained waiting introduces hardness and parameterized boundaries [19, 20].
- Interdiction variants now provide complexity separations between different temporal metrics [10].

**Open gaps with evidence.**

- Approximation and FPT landscapes for multi-criteria temporal routing are thinner than for decision versions [59, 10].
- Interval/link-stream shortest-path complexity is less complete than edge-time sequence models [40, 41].

## Family C: Temporal Connectivity and Substructures

**Why central.** This family generalizes static connectivity design and cut problems to causal connectivity over time, which is where temporal labels become a design resource rather than only an input [4, 1].

**First formalization and refinements.** Early optimization formalizations focus on minimum temporally connected subgraphs and temporal-design complexity [4, 1], with later refinement through separator-based complexity maps and temporal spanner constructions [60, 23, 9].

**Definition forks that change results.**

- Pairwise vs all-pairs temporal connectivity.
- Directed vs undirected temporal labels.
- Connectivity certificates preserving only reachability vs preserving richer path metrics.
- Cut/separator variants under strict vs non-strict and restless semantics [60, 57].

**Canonical techniques.**

- Reductions from set cover/hitting set and cut problems.
- Temporal separator frameworks that expose graph-class tractability [35].
- Sparsification constructions (especially on cliques) for temporal spanners [23].

**What is known.**

- MTC has strong hardness and approximation barriers and can require dense certificates [4].
- Temporal cliques admit significantly sparse spanners; this is one of the clearest positive sparsification results [22, 23, 8].
- Temporal separator/cut problems have detailed parameterized and graph-class complexity maps [60, 35, 57].
- Optimal temporal branchings and temporally spanning subgraph variants now have dedicated optimization algorithms and complexity results, refining the spanning-tree/arborescence branch of the family [11].
- Connectivity augmentation and optimum labeling are active new optimization lines [9, 43].

**Open gaps with evidence.**

- General sparse-spanner guarantees beyond highly structured classes (e.g., cliques) remain limited [23, 8].
- A full approximation/FPT unification across augmentation, labeling, and MTC-style design objectives is not yet available [4, 9, 43].

- Temporal spanning-tree/arborescence formulations are still definition-sensitive, with hardness/relaxation tradeoffs that remain only partially unified [14, 11].

## Family D: Exploration / Traversal / Search

**Why central.** Exploration asks for a temporal walk visiting all vertices (or edges), directly generalizing static exploration/TSP in environments with time-dependent feasibility [29, 56].

**First formalization and refinements.** A core formalization is temporal graph exploration (TGE) [28, 29], with modern variants for non-strict semantics, edge-deficiency, and parameterized formulations [31, 33, 34].

**Definition forks that change results.**

- Offline vs online knowledge of edge availability.
- Vertex-exploration vs edge-exploration objectives.
- Strict vs non-strict time semantics [31].
- Structural restrictions (cycles, bounded edge-deficiency, periodicity) [33].

**Canonical techniques.**

- Hardness reductions from Hamiltonian/traversal problems.
- Timeline compression and dynamic programming on restricted structures.
- Parameterized analysis by graph class and temporal parameters [34].

**What is known.**

- Strong hardness holds even for restrictive temporal settings, with matching algorithmic islands [28, 29].
- Non-strict and k-edge-deficient models have refined complexity boundaries and specialized algorithms [31, 32, 33].
- Parameterized exploration brings FPT-style progress to otherwise hard variants [34].

**Open gaps with evidence.**

- Competitive analysis for online exploration is less complete than offline approximation/parameterized results [31, 34].
- Edge-exploration is substantially younger than vertex-exploration in temporal settings [13].

## Family E: Multistage / Timeline Analogues of NP-hard Problems

**Why central.** This family imports classic NP-hard optimization into temporal timelines where solutions must be valid repeatedly across time and often constrained to change slowly between layers [3, 38].

**First formalization and refinements.** Temporal/sliding-window vertex cover is the canonical early anchor [2, 3], followed by multistage path/cover formulations and timeline-cover extensions [38, 36, 25].

**Definition forks that change results.**

- Sliding-window coverage constraints (**Delta**-window TVC).
- Layerwise feasibility plus inter-layer stability penalties (multistage formulations).

- Activity-interval constraints on vertices/edges [46].

#### **Canonical techniques.**

- Reductions from static covering/path problems to timeline layers.
- Parameterization by window size, lifetime, and budgeted solution changes.
- Approximation by LP relaxations and combinatorial layering arguments.

#### **What is known.**

- Temporal/sliding-window vertex cover has NP-hardness, approximation algorithms, and parameterized results; several restricted classes are now classified [2, 3, 39].
- Temporal matching and temporal coloring each have dedicated hardness and algorithmic results [51, 53].
- Multistage s-t path and multistage vertex cover expose explicit quality-vs-stability tradeoffs [37, 38, 36].
- Timeline-cover formulations continue to receive exact/approximation improvements [25].

#### **Open gaps with evidence.**

- Compared with TVC, multistage analogues of spanning tree/cut problems remain much less unified in temporal-graph-specific models [38, 25].
- Kernelization lower/upper bounds are uneven across multistage families [36, 39].

### **Family F: Counting and Enumeration**

**Why central.** Counting and enumeration quantify the combinatorial size of temporal feasibility spaces and directly support reliability, centrality, and exhaustive design tasks.

**First formalization and refinements.** Dedicated counting-complexity treatment is recent for temporal paths [27], with enumeration work now extending to temporal spanner families [47].

#### **Definition forks that change results.**

- Counting paths vs walks; simple vs non-simple paths.
- Strict/non-strict semantics and waiting constraints.
- Enumeration target: all feasible objects vs minimal/maximal certificates.

#### **Canonical techniques.**

- #P-hardness reductions from static counting problems.
- Parameterized counting with temporal parameters and structural restrictions.
- Output-sensitive enumeration frameworks for temporal substructures.

#### **What is known.**

- Counting temporal paths now has explicit complexity-theoretic and algorithmic treatment [27].
- Enumeration of temporal spanners has emerged as a distinct problem with dedicated algorithms [47].

#### **Open gaps with evidence.**

- Relative to decision/optimization, counting complexity maps are still sparse and non-uniform across models [27].
- Enumeration has concentrated on a few substructures; broad output-polynomial classifications are largely open [47, 23].

## Family G (Cross-cutting): Online and Distributed Temporal Protocols

**Why central.** Temporal-graph feasibility naturally models causal communication in dynamic distributed systems, where future edge availability may be unknown [45, 18].

**First formalization and refinements.** Distributed dynamic-network lower-bound models are foundational [45], and later temporal-broadcast objective taxonomies and online exploration variants connect protocol goals to temporal-graph path semantics [17, 31].

**Definition forks that change results.**

- Adversarial vs stochastic evolution assumptions.
- Knowledge assumptions (offline full timeline vs online revelation).
- Broadcast objective (foremost/fastest/shortest) and whether global synchronization exists [17].

**Canonical techniques.**

- Dynamic-network lower bounds and indistinguishability arguments [45].
- Temporal-broadcast constructions mapped to path metrics [17].
- Competitive analysis inherited from temporal exploration/routing online variants [31].

**Open gaps with evidence.**

- Strong bridges between distributed temporal-network lower bounds and temporal-graph parameterized complexity are still uncommon [45, 34].
- Many protocol results are model-specific, complicating transfer to canonical temporal-graph benchmarks [54, 18].

## Family H: Random/Probabilistic Temporal Graph Theory

**Why central.** This line asks which temporal properties hold typically (a.a.s.) rather than in worst case, complementing hardness-heavy families with threshold phenomena and average-case structural behavior.

**First formalization and refinements.** A key formalization is random simple temporal graphs with independent random edge-presence and edge-label permutations [24], followed by giant-component analyses in random temporal graphs [7].

**Definition forks that change results.**

- What is random: support graph, timestamps, or both.
- Property target: pairwise reachability, source reachability, all-pairs temporal connectivity.
- Regime: sparse vs dense support and how lifetime scales with  $n$ .

**Canonical techniques.**

- Sharp-threshold arguments adapted from random graph theory to temporal constraints.

- Coupling between temporal reachability processes and static percolation-like events.
- Concentration bounds for temporal component sizes.

**What is known.**

- Distinct sharp thresholds exist for pairwise temporal connectivity, single-source temporal connectivity, and all-pairs temporal connectivity [24].
- Giant temporally connected components in random temporal graphs emerge via explicit threshold behavior [7].

**Open gaps with evidence.**

- Approximation/FPT theory and threshold theory are weakly connected; transfer principles are largely absent [4, 24].
- Random-model conclusions are sensitive to timestamp-generation assumptions, and cross-model robustness remains underdeveloped [7].

**Model/Definition Crosswalk (Consolidated Glossary)**

Unified vocabulary in this report	Common alternatives in literature	Semantics (this report)	Notes / representative sources
Temporal graph	Evolving graph, dynamic graph, time-varying graph, link stream	Graph whose edge set is indexed by time labels or intervals	[42, 55, 18]
Snapshot sequence	Layered graph, graph timeline	Sequence ( $G_1, \dots, G_\tau$ ) over discrete time steps	Common in multistage formulations [3, 38]
Contact sequence	Time-labeled edge stream	Set of tuples $(u, v, t[, \lambda])$	Dominant in journey-based path papers [12, 59]
Interval temporal graph	Link stream with durations, interval-label graph	Edge active on interval $[a, b]$ (or multiple intervals)	Important for waiting and traversal-time nuances [40, 41]
Strict temporal path	Journey with strictly increasing times	$t_{i+1} > t_i$	Often default in algorithmic papers [42, 15]
Non-strict temporal path	Weakly increasing journey	$t_{i+1} \geq t_i$	Changes reachability and exploration complexity [31, 15]
Restless path	Bounded-wait path	Waiting on vertices bounded by parameter $\Delta$	Central in waiting-constrained complexity [20, 57]
Foremost path	Earliest-arrival path	Minimize arrival time given departure/start condition	[12, 59]
Fastest path	Minimum-duration path	Minimize arrival - departure	[12, 17]
Temporal shortest path	Minimum-hop temporal path	Minimize hop count or edge cost under temporal feasibility	[58, 59]

Unified vocabulary in this report	Common alternatives in literature	Semantics (this report)	Notes / representative sources
Temporal connectivity	Temporal all-pairs reachability	Every ordered/unordered pair temporally connected	[4, 1]
Temporal spanner	Temporal connectivity certificate	Sparse temporal subgraph preserving chosen temporal property	[23, 47]
Temporal separator	Temporal cut, $(s, z)$ temporal separator	Vertex/edge set whose deletion destroys temporal reachability	[60, 35]
Sliding-window TVC	Delta-TVC, temporal vertex cover	Every edge active in each length-Delta window is covered	[2, 3]
Multistage optimization	Timeline optimization	Sequence of solutions with per-layer feasibility and transition constraints	[38, 36, 25]
Reachability-graph realization	Temporal realization/inference	Construct temporal labels matching a target reachability relation	[30, 16, 48]

## Conclusion and Taxonomy-at-a-Glance

The theoretical temporal-graph community has converged on a stable **core** (path semantics, shortest-path variants, temporal connectivity design/cuts, exploration, and temporalized NP-hard classics) and a clearly visible **frontier** (realization/inference, interdiction, counting/enumeration, and stronger online/distributed integration). Adjacent probabilistic work on random temporal structures also informs the frontier, especially for phase-transition intuition behind sparse certificates and connectivity thresholds [7].

A robust reading strategy for new papers is:

1. Identify the temporal model and journey semantics first.
2. Identify which objective family is being optimized.
3. Check whether results are offline, online, or distributed-adversarial.
4. Map to the nearest known hardness/FPT/approximation boundary in the family taxonomy.

### Taxonomy at a glance

Family	Core/Niche status	Current maturity
Reachability, closure, realization	Core (with niche realization branch)	Mature foundations, active refinements
Shortest-path/routing variants	Core	Mature objectives, active constrained/interdiction work
Connectivity/substructures (MTC, spanners, separators, augmentation)	Core	Strong hardness + growing constructive algorithms
Exploration/traversal/search	Core	Mature hardness, expanding FPT/online directions
Multistage/timeline NP-hard analogues	Core-fast-growing	TVC mature; other formulations less unified
Counting/enumeration	Niche-fast-growing	Early complexity maps, many open classifications

Family	Core/Niche status	Current maturity
Online/distributed temporal protocols	Cross-cutting niche	Deep lower-bound tradition, limited unification with mainstream taxonomy
Random/probabilistic temporal graph theory	Frontier niche	Sharp-threshold progress, limited integration with worst-case complexity maps

## References

- [1] Elena Akrida, George B. Mertzios, Paul G. Spirakis, and Viktor Zamaraev. The complexity of optimal design of temporally connected graphs. *Theory of Computing Systems*, 61(3):907–944, 2017.
- [2] Elena Akrida, George B. Mertzios, Paul G. Spirakis, and Viktor Zamaraev. Temporal vertex cover with a sliding time window. In *Proceedings of ICALP*, 2018.
- [3] Elena Akrida, George B. Mertzios, Paul G. Spirakis, and Viktor Zamaraev. Temporal vertex cover with a sliding time window. *Journal of Computer and System Sciences*, 107:108–123, 2020.
- [4] Kyriakos Axiotis and Dimitris Fotakis. On the size and the approximability of minimum temporally connected subgraphs. In *Proceedings of ICALP*, 2016.
- [5] Stefan Balev, Eric Sanlaville, and Jason Schoeters. Temporally connected components in temporal graphs. *Theoretical Computer Science*, 1000:114518, 2024.
- [6] Stefan Balev, Eric Sanlaville, and Céline Toullalan. Shortest temporal exploration problem. In *Proceedings of SAND (Brief Announcement)*, 2025.
- [7] Ruben Becker, Arnaud Casteigts, Pierluigi Crescenzi, Renars Kodric, Malte Renken, Arnaud Raskin, and Viktor Zamaraev. Giant components in random temporal graphs. In *Proceedings of APPROX/RANDOM*, pages 14:1–14:19, 2023.
- [8] Ruben Becker, Arnaud Casteigts, Pierluigi Crescenzi, Renars Kodric, Malte Renken, Arnaud Raskin, and Viktor Zamaraev. How to reduce temporal cliques to find sparse spanners. *arXiv preprint arXiv:2408.08370*, 2024.
- [9] Nicolas Bellitto, Eudes Bouton, Guillaume Popper, and Laurent Escoffier. Temporal connectivity augmentation. In *Proceedings of SAND*, 2025.
- [10] Anna-Lena Boeckmann, Clemens Thielen, and Janosch Wittmann. On the complexity of the temporal shortest path interdiction problem. In *Proceedings of SAND*, 2023.
- [11] Daniele Bubboloni, Arnaud Casteigts, Flavien Corsini, Malte Renken, and Viktor Zamaraev. On computing optimal temporal branchings and spanning subgraphs. *Journal of Computer and System Sciences*, 148:103596, 2025.
- [12] Binh-Minh Bui-Xuan, Afonso Ferreira, and Alain Jarry. Computing shortest, fastest, and foremost journeys in dynamic networks. *International Journal of Foundations of Computer Science*, 14(2):267–285, 2003.
- [13] Benjamin Bumpus and Kitty Meeks. Edge exploration of temporal graphs. *Algorithmica*, 85(6):1709–1742, 2023.
- [14] Arnaud Casteigts and Flavien Corsini. In search of the lost tree: Hardness and relaxation of spanning trees in temporal graphs. In *Proceedings of SIROCCO*, 2024.
- [15] Arnaud Casteigts, Flavien Corsini, and Bivas Mitra Sarkar. Simple, strict, proper, happy: A study of reachability in temporal graphs. *Theoretical Computer Science*, 1000:114520, 2024.
- [16] Arnaud Casteigts, Sebastian D'oring, and Fabian Morawietz. Realization of temporally connected graphs based on degree sequences. In *Proceedings of ISAAC*, 2025.

- [17] Arnaud Casteigts, Paola Flocchini, Bernard Mans, and Nicola Santoro. Shortest, fastest, and foremost broadcast in dynamic networks. *International Journal of Foundations of Computer Science*, 26(4):499–519, 2015.
- [18] Arnaud Casteigts, Paola Flocchini, Walter Quattrociocchi, and Nicola Santoro. Time-varying graphs and dynamic networks. In *Dynamics On and Of Complex Networks, Volume 2*. Springer, 2013.
- [19] Arnaud Casteigts, Bodo Himmel, Hendrik Molter, and Philipp Zschoche. Finding temporal paths under waiting time constraints. In *Proceedings of ISAAC*, 2020.
- [20] Arnaud Casteigts, Bodo Himmel, Hendrik Molter, and Philipp Zschoche. Finding temporal paths under waiting time constraints. *Algorithmica*, 83(9):2754–2802, 2021.
- [21] Arnaud Casteigts, Fabian Morawietz, and Petra Wolf. Distance to transitivity in temporal graphs. In *Proceedings of MFCS*, 2024.
- [22] Arnaud Casteigts, Jason Peters, and Jason Schoeters. Temporal cliques admit sparse spanners. In *Proceedings of ICALP*, 2019.
- [23] Arnaud Casteigts, Jason Peters, and Jason Schoeters. Temporal cliques admit sparse spanners. *Journal of Computer and System Sciences*, 121:1–17, 2021.
- [24] Arnaud Casteigts, Mikhail Raskin, Malte Renken, and Viktor Zamaraev. Sharp thresholds in random simple temporal graphs. In *Proceedings of FOCS*, pages 319–326, 2021.
- [25] Riccardo Dondi and Alexandru Popa. Exact and approximation algorithms for covering timeline in temporal graphs. *Annals of Operations Research*, 2025.
- [26] Sebastian Doring and Hendrik Molter. Simple, strict, proper, and directed reachability in temporal graphs. In *Proceedings of ISAAC*, 2025.
- [27] Jessica Enright, Kitty Meeks, and Hendrik Molter. Counting temporal paths. *Algorithmica*, 2025.
- [28] Thomas Erlebach, Michael Hoffmann, and Frank Kammer. On temporal graph exploration. In *Proceedings of ICALP*, 2015.
- [29] Thomas Erlebach, Michael Hoffmann, and Frank Kammer. On temporal graph exploration. *Journal of Computer and System Sciences*, 119:1–18, 2021.
- [30] Thomas Erlebach, Othon Michail, and Fabian Morawietz. Recognizing and realizing temporal reachability graphs. In *Proceedings of ESA*, 2025.
- [31] Thomas Erlebach and Jacob T. Spooner. Non-strict temporal exploration. In *Proceedings of SIROCCO*, 2020.
- [32] Thomas Erlebach and Jacob T. Spooner. Exploration of k-edge-deficient temporal graphs. In *Proceedings of WADS*, 2021.
- [33] Thomas Erlebach and Jacob T. Spooner. Exploration of k-edge-deficient temporal graphs. *Acta Informatica*, 59(8):955–989, 2022.
- [34] Thomas Erlebach and Jacob T. Spooner. Parameterized temporal exploration problems. *Journal of Computer and System Sciences*, 137:66–86, 2023.
- [35] Till Fluschnik, Hendrik Molter, Rolf Niedermeier, Malte Renken, and Philipp Zschoche. Temporal graph classes: A view through temporal separators. *Theoretical Computer Science*, 806:197–218, 2020.
- [36] Till Fluschnik, Rolf Niedermeier, Valentin Rohm, and Philipp Zschoche. Multistage vertex cover. *Theory of Computing Systems*, 66(1):331–363, 2022.
- [37] Till Fluschnik, Rolf Niedermeier, Maximilian Schubert, and Philipp Zschoche. Multistage s-t path: Confronting similarity with dissimilarity. In *Proceedings of ISAAC*, 2020.

- [38] Till Fluschnik, Rolf Niedermeier, Maximilian Schubert, and Philipp Zschoche. Multistage s-t path: Confronting similarity with dissimilarity. *Algorithmica*, 85(1):38–75, 2023.
- [39] Thekla Hamm, Nina Klobas, George B. Mertzios, and Paul G. Spirakis. The complexity of temporal vertex cover in small-degree graphs. In *Proceedings of AAAI*, 2022.
- [40] Pankaj K. Jain and Sartaj Sahni. Foremost walks and paths in interval temporal graphs. *Algorithms*, 15(4):121, 2022.
- [41] Pankaj K. Jain and Sartaj Sahni. Polynomial time algorithms for shortest paths in weighted interval temporal graphs. *Algorithms*, 17(2):70, 2024.
- [42] David Kempe, Jon M. Kleinberg, and Amit Kumar. Connectivity and inference problems for temporal networks. *Journal of Computer and System Sciences*, 64(4):820–842, 2002.
- [43] Nina Klobas, George B. Mertzios, Hendrik Molter, and Paul G. Spirakis. The complexity of computing optimum labelings for temporal connectivity. *Journal of Computer and System Sciences*, 147:103605, 2024.
- [44] Nina Klobas, George B. Mertzios, Hendrik Molter, and Paul G. Spirakis. Temporal graph realization from fastest paths. In *Proceedings of SAND*, 2024.
- [45] Fabian Kuhn, Nancy Lynch, and Rotem Oshman. Distributed computation in dynamic networks. In *Proceedings of STOC*, pages 513–522, 2010.
- [46] Ryohei Kurita, Alessandro Marino, Jason Schoeters, and Takeaki Uno. Partial temporal vertex cover with bounded activity intervals. In *Proceedings of SAND*, 2024.
- [47] Ryohei Kurita, Alessandro Marino, Jason Schoeters, and Takeaki Uno. Spanner enumeration for temporal graphs. In *Proceedings of SAND*, 2025.
- [48] George B. Mertzios, Hendrik Molter, Fabian Morawietz, and Paul G. Spirakis. Temporal graph realization from fastest paths. *Theoretical Computer Science*, 2025.
- [49] George B. Mertzios, Hendrik Molter, Fabian Morawietz, and Paul G. Spirakis. Temporal graph realization with bounded stretch. In *Proceedings of MFCS*, 2025.
- [50] George B. Mertzios, Hendrik Molter, Rolf Niedermeier, Viktor Zamaraev, and Philipp Zschoche. Computing maximum matchings in temporal graphs. In *Proceedings of STACS*, 2020.
- [51] George B. Mertzios, Hendrik Molter, Rolf Niedermeier, Viktor Zamaraev, and Philipp Zschoche. Computing maximum matchings in temporal graphs. *Journal of Computer and System Sciences*, 137:1–27, 2023.
- [52] George B. Mertzios, Hendrik Molter, and Viktor Zamaraev. Sliding window temporal graph coloring. In *Proceedings of AAAI*, 2019.
- [53] George B. Mertzios, Hendrik Molter, and Viktor Zamaraev. Sliding window temporal graph coloring. *Journal of Computer and System Sciences*, 120:97–115, 2021.
- [54] George Michail, Ioannis Chatzigiannakis, and Paul G. Spirakis. Causality, influence, and computation in possibly disconnected synchronous dynamic networks. In *Proceedings of OPODIS*, 2012.
- [55] Othon Michail. An introduction to temporal graphs: An algorithmic perspective. *Internet Mathematics*, 12(4):239–280, 2016.
- [56] Othon Michail and Paul G. Spirakis. Traveling salesman problems in temporal graphs. *Theoretical Computer Science*, 634:1–23, 2016.
- [57] Hendrik Molter. Restless temporal (s, z)-separation. *Information Processing Letters*, 178:106305, 2022.
- [58] Hao Wu, James Cheng, Silu Huang, Yiping Ke, Yanyan Lu, and Yufei Xu. Path problems in temporal graphs. *Proceedings of the VLDB Endowment*, 7(9):721–732, 2014.

- [59] Hao Wu, James Cheng, Silu Huang, Yiping Ke, Yanyan Lu, and Yufei Xu. Path problems in temporal graphs. *IEEE Transactions on Knowledge and Data Engineering*, 28(9):2225–2238, 2016.
- [60] Philipp Zschoche, Till Fluschnik, Hendrik Molter, and Rolf Niedermeier. The complexity of finding small separators in temporal graphs. *Journal of Computer and System Sciences*, 107:72–92, 2020.