

Named Temporal Graph Classes in the Literature

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Abstract

This note inventories named temporal graph classes that have explicit definitions in the algorithmic temporal-graph literature. The scope is finite discrete-time temporal graphs. The inventory is intentionally conservative: it includes named temporal restrictions and named hybrid classes such as temporal interval graphs, but it does not pretend that there is a finite list once one also counts every schema of the form “each layer belongs to a static class \mathcal{C} ” or every bounded-parameter family. Those open-ended schemas are discussed separately.

1 Scope and Model

We work with a finite temporal graph $\mathcal{G} = (V, E, \lambda)$, where V is the vertex set, $E \subseteq \binom{V}{2}$ is the footprint edge set, and $\lambda : E \rightarrow 2^{[\tau]} \setminus \{\emptyset\}$ assigns each edge its set of presence times. Equivalently, the t -th layer is $G_t = (V, E_t)$ with

$$E_t := \{e \in E \mid t \in \lambda(e)\}.$$

We write $G_{\downarrow} = (V, E)$ for the footprint. A temporal path is a sequence of time-edges whose underlying edges form a static path and whose times are non-decreasing; it is *strict* if the times are strictly increasing [KKK02; Mic16].

The term “temporal graph class” is unfortunately overloaded. In the literature, it may mean at least three different things.

- A named restriction on temporal evolution, such as p -monotone or λ -steady [Flu+20].
- A named hybrid class obtained by combining the temporal model with a static graph class, such as temporal interval graphs or order-preserving temporal unit interval graphs [Flu+20; Her+22; Her+23].
- A schema that generates infinitely many classes, for example “temporal \mathcal{C} -graphs” for arbitrary static classes \mathcal{C} , or families of graphs with bounded temporal neighborhood diversity or bounded temporal cliquewidth [Enr+24].

The main inventory below covers the first two categories. The third is discussed in Section 7. I also exclude the separate distributed *time-varying graph* hierarchy, which has its own class system based on recurrence and periodicity assumptions [Cas+13].

2 Inventory Table

Class	Defining idea	Standard source
p -monotone temporal graph	The lifetime can be partitioned into p contiguous blocks, each monotone by edge-set inclusion.	[Flu+20]
Grounded temporal graph	Some layer contains all other layers’ edge sets.	[Flu+20]

Class	Defining idea	Standard source
p -periodic temporal graph	Layers repeat with period p .	[Flu+20; Cas+13]
T -interval connected temporal graph	Every window of T consecutive layers has a connected edge-set intersection.	[KLO10; Flu+20]
λ -steady temporal graph	Consecutive layers differ by at most λ edge insertions or deletions.	[Flu+20]
Temporal interval graph	Every layer is an interval graph.	[Her+23]
Temporal unit interval graph	Every layer is a unit interval graph.	[Her+22; Her+23]
Order-preserving temporal interval graph	All layers admit interval models consistent with one common vertex ordering.	[Flu+20; Her+23]
Order-preserving temporal unit interval graph	All layers admit unit-interval models consistent with one common vertex ordering.	[Flu+20; Her+22]
Temporally connected graph (TC)	Every ordered pair of vertices is connected by a temporal path.	[KKK02; CCS24]
Recurrently temporally connected graph (TC^R)	Infinite-lifetime analogue of TC , where temporal connectivity occurs infinitely often.	[CCS24]
Proper temporal graph	Incident edges never share a time label.	[CCS24]
Simple temporal graph	Each edge has exactly one presence time.	[CCS24]
Happy temporal graph	Both proper and simple.	[CCS24]
Temporally transitively orientable graph	Admits an orientation satisfying temporal transitivity for non-decreasing time pairs.	[Mer+21]
Strictly temporally transitively orientable graph	Admits an orientation satisfying temporal transitivity for strictly increasing time pairs.	[Mer+21]

3 Evolution-Restricted Classes

p -monotone temporal graphs. Fluschnik et al. define \mathcal{G} to be p -monotone if p is the smallest number such that there are $1 = i_1 < i_2 < \dots < i_{p+1} = \tau$ and, for each block $[i_\ell, i_{\ell+1}]$, either $E_j \subseteq E_{j+1}$ for all $i_\ell \leq j < i_{\ell+1}$, or $E_j \supseteq E_{j+1}$ for all $i_\ell \leq j < i_{\ell+1}$ [Flu+20]. Thus 1-monotonicity means globally monotone growth or decay, while larger p allows a bounded number of trend reversals.

Grounded temporal graphs. The same paper isolates the special case in which one layer contains all others, that is, there exists $i \in [\tau]$ such that $E_t \subseteq E_i$ for every t . The paper uses the term *grounded* for this situation in its summary discussion [Flu+20]. This is one of the few cases where temporal (s, z) -separation becomes trivial.

p -periodic temporal graphs. Following edge-periodicity notions already present in the time-varying graph literature [Cas+13], Fluschnik et al. study p -periodic temporal graphs, where layer structure repeats with period p . In finite lifetime notation, the defining condition is $E_t = E_{t+p}$ whenever $t + p \leq \tau$; equivalently, \mathcal{G} is obtained by repeating a temporal graph of lifetime p [Flu+20].

T -interval connected temporal graphs. Kuhn, Lynch, and Oshman introduced T -interval connectivity in dynamic networks, and Fluschnik et al. import the notion into temporal graph algorithmics [KLO10; Flu+20]. A temporal graph is T -interval connected if, for every $t \in [\tau - T + 1]$, the static graph

$$\left(V, \bigcap_{i=t}^{t+T-1} E_i\right)$$

is connected. Informally, every window of T consecutive layers has a connected “persistent core”. Fluschnik et al. refer to this class informally as “consecutively connected” in their overview [Flu+20].

λ -steady temporal graphs. A temporal graph is λ -steady if λ is the smallest number such that

$$|E_t \Delta E_{t+1}| \leq \lambda$$

for every $t \in [\tau - 1]$, where Δ denotes symmetric difference [Flu+20]. This captures bounded per-step volatility and is one of the cleanest evolution-only temporal classes in the literature.

4 Interval-Layer and Order-Preservation Classes

Temporal interval graphs. Hermelin, Itzhaki, Molter, and Niedermeier define temporal interval graphs as temporal graphs whose every layer is an interval graph [Her+23]. This is the direct layerwise lift of interval graphs to the temporal setting.

Temporal unit interval graphs. The same authors, in their earlier SAND paper and subsequent journal version, focus on temporal unit interval graphs, where every layer is a unit interval graph [Her+22; Her+23]. This is a proper subclass of temporal interval graphs and was the first interval-based temporal graph class studied systematically in this line of work.

Order-preserving temporal interval graphs. Order preservation was introduced by Fluschnik et al. for temporal unit interval graphs and then generalized explicitly to temporal interval graphs by Hermelin et al. [Flu+20; Her+23]. A temporal interval graph is order-preserving if there exists a vertex ordering $<_V$ such that each layer admits an interval model whose left endpoints and right endpoints are both ordered according to $<_V$ [Her+23]. The point is not just that every layer is interval, but that all layers can be represented with one common combinatorial ordering.

Order-preserving temporal unit interval graphs. For temporal unit interval graphs, the same common-order idea yields the class of order-preserving temporal unit interval graphs [Flu+20; Her+22]. Hermelin et al. describe the condition informally as requiring “the intervals of every layer to obey a common ordering” [Her+22]. Fluschnik et al. also define the related *shuffle number*, while Hermelin et al. define the *order-preserving vertex deletion* distance; those are parameters, not additional standalone classes.

5 Reachability- and Labeling-Based Classes

Temporally connected graphs (TC). The basic reachability class is the class of temporally connected graphs: every ordered pair of vertices is connected by a temporal path. Equivalently, the temporal reachability graph $\mathcal{R}(\mathcal{G})$ is a complete digraph [KKK02; CCS24]. This class is older than most explicitly named temporal graph classes and is arguably the default global connectivity notion in the area.

Recurrently temporally connected graphs (TC^R). Casteigts, Corsini, and Sarkar explicitly refer to the infinite-lifetime analogue TC^R , where temporal connectivity is achieved recurrently, that is, infinitely often [CCS24]. This belongs more naturally to the time-varying-graph viewpoint than to finite-lifetime algorithmics, but it is a named class and should be recorded.

Proper temporal graphs. A temporal graph is *proper* if $\lambda(e) \cap \lambda(e') = \emptyset$ whenever e and e' are incident to a common vertex [CCS24]. In other words, adjacent edges never coexist at the same time. The 2024 reachability paper treats properness as a graph-class restriction rather than merely a path-semantic option.

Simple temporal graphs. A temporal graph is *simple* if λ is single-valued, that is, every edge has exactly one presence time [CCS24]. Simple temporal graphs already play a role in older papers, but the 2024 paper makes the class explicit and studies its expressivity systematically.

Happy temporal graphs. The same paper defines a temporal graph to be *happy* if it is both proper and simple [CCS24]. Although the name is informal, it is not a joke definition: the paper argues that happy temporal graphs are expressive enough to transfer several negative results and should be considered a canonical restricted target class.

6 Temporal Orientation Classes

Temporally transitively orientable graphs. Mertzios, Molter, Renken, Spirakis, and Zschoche introduce temporal transitive orientations [Mer+21]. An orientation of a temporal graph is temporally transitive if whenever $u \xrightarrow{t_1} v$ and $v \xrightarrow{t_2} w$ with $t_2 \geq t_1$, then there is also $u \xrightarrow{t_3} w$ with $t_3 \geq t_2$. The corresponding class consists of temporal graphs admitting such an orientation.

Strictly temporally transitively orientable graphs. The same paper defines the strict variant by requiring the implication only when $t_2 > t_1$, matching strict temporal paths [Mer+21]. The two classes behave very differently: recognition of temporal transitive orientability is polynomial-time, whereas recognition of the strict variant is NP-hard [Mer+21].

7 Open-Ended Class Schemas

The literature also contains several sources of temporal graph classes that do *not* form a finite list.

Layerwise lifts of static classes. Once one allows definitions of the form “every layer G_t belongs to a static graph class \mathcal{C} ”, one immediately gets an infinite class schema: temporal interval graphs, temporal unit interval graphs, temporal chordal graphs, temporal planar graphs, and so on. Only a few such lifts have been given dedicated study so far; interval-based lifts are the most visible examples [Flu+20; Her+22; Her+23].

Bounded-parameter families. Similarly, many papers define structural parameters whose bounded-value instances are themselves graph classes. Examples include bounded shuffle number and bounded order-preserving vertex deletion in the interval literature [Flu+20; Her+23], and bounded temporal cliquewidth, bounded temporal modular-width, and bounded temporal neighborhood diversity in the dense-temporal-graph line [Enr+24]. These are valid temporal graph classes in the formal sense, but not isolated named classes with a finite inventory.

Distributed time-varying graph hierarchies. The dynamic-network literature contains a separate hierarchy of classes based on recurrence, periodicity, and connectivity assumptions over possibly infinite executions [Cas+13]. Those classes are important, but mixing that hierarchy into the present inventory without care would conflate two adjacent but distinct literatures.

8 Conclusion

Within the narrow but defensible scope of *named* temporal graph classes used in the finite-time algorithmic literature, the list is still surprisingly short. Most explicitly defined classes fall into four families:

- evolution-only restrictions such as p -monotonicity, periodicity, interval connectivity, and steadiness;
- per-layer interval restrictions and their order-preserving variants;
- reachability- or labeling-based restrictions such as TC , proper, simple, and happy;
- orientation-based restrictions such as temporal transitive orientability.

If one also counts every layerwise lift and every bounded-parameter family, then there is no finite exhaustive list. Any claim of exhaustiveness therefore has to say *within which scope*.

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